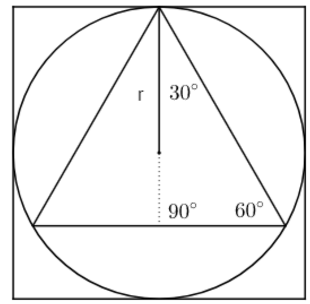
1. D
2. E
3. A
4. E
5. A
6. E
7. A
8. C
9. B
10. B
11. C
12. A
13. B
14. D
15. B
16. C
17. B
18. E
19. D
20. E
21. D
22. B
23. C
24. B
25. D
26. C
27. B
28. A
29. C
30. E

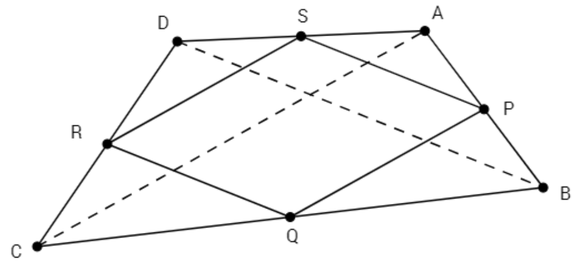
1. **D.** The given statements indicate that $A = k_1B^2$, $B = \frac{k_2}{C^3}$, and $C = \frac{k_3}{D}$ for some fixed values of k_1, k_2, and k_3. Combining these we find $A = k_1B^2 = k_1\left(\frac{k_2}{C^3}\right)^2 = \frac{k_1k_2}{C^6} = \frac{k_1k_2}{(k_3/D)^6} = \frac{k_1k_2}{k_3^6}D^6$. Plugging the given values into the rightmost and leftmost portions of that string of equalities we see $2 = \frac{k_1k_2}{k_3^6}(1)^6$ which allows us to conclude $\frac{k_1k_2}{k_3^6} = 2$. That means $A = 2D^6$. Plugging in $D = 2$ we find $A = 2(2)^6 = 128$.
2. **E.** Based on the point (a, b) being on the graph of $y = x^2$, we know that $b = a^2$. So, we rewrite the second parabola as $y = a^2x^2 + ax + 4$. The vertex of this parabola will occur at an x-value of $x = \frac{-a}{2(a^2)} = \frac{-1}{2a}$. The y-value that this corresponds with is $y = a^2\left(\frac{-1}{2a}\right)^2 + a\left(\frac{-1}{2a}\right) + 4 = \frac{a^2}{4a^2} - \frac{a}{2a} + 4 = \frac{1}{4} - \frac{1}{2} + 4 = \frac{15}{4}$. So, the point (c, d) can be expressed as $\left(\frac{-1}{2a}, \frac{15}{4}\right)$. Since $c + d = 4$, we see that $-\frac{1}{2a} + \frac{15}{4} = 4$ which leads to $-\frac{1}{2a} = \frac{1}{4}$. We conclude that $a = -2$. So, $b = (-2)^2 = 4$. $|a + b| = |-2 + 4| = 2$.
3. **A.** There are ${}_{10}C_5 = \frac{10!}{5!5!} = 252$ ways to pick 5 animals to be on a team together. The unpicked animals will form the second team. Of the 252 combinations, 2 of them are illegal (the all cats selection and the all dogs selection). So, there are 250 ways to form a legal team. However, this double counts every match-up as it counts a match up once when a set of five animals is all selected and again when the complement of that set is selected. So, we divide 250 by 2 to find that there are actually 125 ways to divide the ten animals into two basketball teams.
4. **E.** $\left|\big||x-2|-2\big|-2\right| - 2 = 0$ implies $\left|\big||x-2|-2\big|-2\right| = 2$.
   $\big||x-2|-2\big| - 2 = -2$ or $\big||x-2|-2\big| - 2 = 2$
   $\big||x-2|-2\big| = 0$ or $\big||x-2|-2\big| = 4$
   $|x-2| - 2 = 0$ or $|x-2| - 2 = -4$ or $|x-2| - 2 = 4$
   $|x-2| = 2$ or $|x-2| = -2$ or $|x-2| = 6$ (the middle equality is extraneous)
   $x - 2 = -2$ or $x - 2 = 2$ or $x - 2 = -6$ or $x - 2 = 6$
   $x = 0$ or $x = 4$ or $x = -4$ or $x = 8$.
   $0 + 4 + (-4) + 8 = 8$
5. **A.** The remainder requested will be twice the remainder of $\frac{1}{2}f(x) \div \left(x - \frac{1}{2}\right)$.
   The remainder of $\frac{1}{2}f(x) \div \left(x - \frac{1}{2}\right)$ is $\frac{1}{2}f\left(\frac{1}{2}\right) = \frac{1}{2}\left(\left(\frac{1}{2}\right)^{10} - \left(\frac{1}{2}\right)^9 + \left(\frac{1}{2}\right)^8 - \cdots - \left(\frac{1}{2}\right) + 1\right)$
   We use the finite geometric series formula to condense this into $\frac{1}{2}\left(\frac{a_1(1-r^n)}{1-r}\right) = \frac{1}{2}\left(\frac{\frac{1}{2^{10}}\left(1-(-2)^{11}\right)}{1-(-2)}\right) = \frac{1}{2^{11}}\left(\frac{1+2048}{3}\right) = \frac{2049}{3\times2^{11}} = \frac{683}{2^{11}}$. The question asked for the remainder of $f(x) \div (2x - 1)$ which is twice the remainder we just found. $2 \times \frac{683}{2^{11}} = \frac{683}{1024}$.

6. **E.** The radius of the circle will be half the side length of the square. So, if we denote the circles radius with r we may say $S = (2r)^2 = 4r^2$. In an equilateral triangle, the circumcenter (the center of the circle) is also the centroid. Since centroids divide medians into segments in a $2:1$ ratio, we realize that the radius of the circle is $\frac{2}{3}$ the height of the triangle. The height of the equilateral triangle is $\frac{3}{2}r$. Solving a $30° - 60° - 90°$ triangle, we realize the base of the triangle has length $r\sqrt{3}$. So, the area of the triangle is $T = \frac{1}{2}\left(r\sqrt{3}\right)\left(\frac{3}{2}r\right) = \frac{3\sqrt{3}r^2}{4}$. Finally, $\frac{T}{S} = \frac{3\sqrt{3}r^2}{4} \div (4r^2) = \frac{3\sqrt{3}}{16}$.



7. **A.** $f(x) = \frac{3x^3-2x^2-4x}{x^2-1} = 3x - 2 + \frac{-x-2}{x^2-1}$ approaches the oblique asymptote $y = 3x - 2$ obtaining the exact value of this asymptote only when $\frac{-x-2}{x^2-1} = 0$. This happens when $x = -2$.
8. **C.** The domain is restricted by the log which requires its argument x^2 to be positive. $x^2 > 0$ means $x \neq 0$. The domain is also restricted by the square root which requires $|x-6| - 2 \geq 0$ (can't take the square root of a negative) and by the quotient which requires $\sqrt{|x-6|-2} \neq 0$ (can't divide by zero). These two restrictions may be combined and expressed as $|x-6| - 2 > 0$. This means $x \notin \{4,5,6,7,8\}$. These five integers are not in the domain of $f(x)$ and 0 is not in the domain of $f(x)$. Thus, six integers are not in the domain.
9. **B.** Let $x = 1 + \frac{1}{1+\frac{1}{1+\cdots}}$. This may be rewritten as $x = 1 + \frac{1}{x}$ which rearranges to $\frac{x^2-x-1}{x} = 0$. Applying the quadratic formula to the numerator, we solve $x = \frac{1-\sqrt{5}}{2}$ or $x = \frac{1+\sqrt{5}}{2}$. Referring back to the way we originally defined x we see that x cannot be negative, so the first possible solution is extraneous. We conclude $x = \frac{1+\sqrt{5}}{2}$. So, $\left(1 + \frac{1}{1+\frac{1}{1+\cdots}}\right)^2 = \left(\frac{1+\sqrt{5}}{2}\right)^2 = \frac{1+2\sqrt{5}+5}{4} = \frac{3+\sqrt{5}}{2}$.

10. **B.** This question is solved quickly by applying Varignon's Theorem. Without knowledge of this theorem, one may solve it by subtracting off the area not included in the new quadrilateral. We will denote the area of polygons by putting the name of the polygon in parentheses.



$$\begin{aligned}(PQRS) &= (ABCD) - (PBQ) - (RDS) - (QCR) - (SAP)\\ &= (ABCD) - \tfrac{1}{4}(ABC) - \tfrac{1}{4}(CDA) - \tfrac{1}{4}(BCD) - \tfrac{1}{4}(DAB)\\ &= (ABCD) - \tfrac{1}{4}[(ABC) + (CDA)] - \tfrac{1}{4}[(BCD) + (DAB)]\\ &= (ABCD) - \tfrac{1}{4}(ABCD) - \tfrac{1}{4}(ABCD)\\ &= \tfrac{1}{2}(ABCD) = \tfrac{1}{2} \times 44 = 22\end{aligned}$$

11. **C.** $f(x) = 2x^3 + 3x^2 - 8x - 12 = x^2(2x+3) - 4(2x+3) = (x^2-4)(2x+3) = (x-2)(x+2)(2x+3)$. The zeros are 2, -2, and $-\frac{3}{2}$. $|r_1| + |r_2| + |r_3| = |2| + |-2| + \left|-\frac{3}{2}\right| = 2 + 2 + \frac{3}{2} = \frac{11}{2}$.

12. **A.** There are ${}_6C_3$ choices for the three numbers to be paired up. For each set of 3 numbers, there are ${}_6C_2$ choices for which dice will show the smallest number. That leaves ${}_4C_2$ choices for which dice will show the largest number. The middle number must be on the remaining dice. So, we have found that of the 6^6 possible outcomes of rolling six dice, ${}_6C_3 \times {}_6C_2 \times {}_4C_2$ result in three distinct pairs of numbers. The probability we seek is $\frac{{}_6C_3 \times {}_6C_2 \times {}_4C_2}{6^6} = \frac{\frac{6!}{3!3!} \times \frac{6!}{4!2!} \times \frac{4!}{2!2!}}{6^6} = \frac{6!6!}{3!3!2!2!2!\times 6^6} = \frac{(6\times5\times4)(6\times5\times4)}{8\times6^6} = \frac{5\times5\times2}{6^4} = \frac{25}{3\times6^3} = \frac{25}{648}$.

13. **B.** Represent Marissa's enclosure with the triangle ABC. Let BC be the side formed by the river and let x be the length of fencing Marissa will use for side AB. Orienting the triangle with side AB as the base, the area of the enclosure is $\frac{1}{2}(x)(h)$ where h is the height of the altitude dropped from point C. By varying how much of the river is used, Marissa can choose an enclosure with any value of h between 0 and the length of AC. Since x feet of fencing are used to form side AB, the maximum possible length of side AC (and therefore the maximum possible value of h) is $40 - x$. To maximize the area of her enclosure $\frac{1}{2}(x)(h)$, Marissa will maximize the value of h. That is, she will make h equal to $40 - x$. Then, the area of her enclosure is

$\frac{1}{2}(x)(40-x) = -\frac{1}{2}x^2 + 20x$. This area function is quadratic, so it is maximized at its vertex. It is maximized at $x = \frac{-20}{2\left(-\frac{1}{2}\right)} = 20$ which corresponds with an area of $\frac{1}{2}(20)(40-20) = 200$.

14. **D.** If the graphs intersect at (A, B), then $B = \ln A$ and $B = \frac{1}{A}$. So, $\ln A = \frac{1}{A}$. This rearranges to $A \ln A = 1$ which implies $\ln A^A = 1$. Converting this to exponential form, we see $A^A = e^1 = e$.

15. **B.** $\sum_{i=5}^{24} \ln\left(1 + \frac{1}{i+5}\right) = \ln\left(1 + \frac{1}{10}\right) + \ln\left(1 + \frac{1}{11}\right) + \cdots + \ln\left(1 + \frac{1}{28}\right) + \ln\left(1 + \frac{1}{29}\right)$

$$= \ln\left(\frac{11}{10}\right) + \ln\left(\frac{12}{11}\right) + \ln\left(\frac{13}{12}\right) + \cdots + \ln\left(\frac{29}{28}\right) + \ln\left(\frac{30}{29}\right)$$

$$= \ln\left(\frac{11}{10} \times \frac{12}{11} \times \frac{13}{12} \times \ldots \times \frac{29}{28} \times \frac{30}{29}\right) = \ln\left(\frac{30}{10}\right) = \ln(3)$$

16. **C.** Let's first count how many three letter words may be formed that use both of the I's. If we form a three letter word that includes both of the I's in WINDIGO, then there are 5 choices of which letter to pair with the I's and 3 choices of where that letter should appear in the word (which defines the arrangement completely). So, there are $5 \times 3 = 15$ three letter words that include both I's. Now, let's count how many three letters words may be formed that do not include both I's. This is equivalent to asking how many three letter words may be formed from the letters in WINDGO. The answer to that ${}_6P_3 = \frac{6!}{3!} = 6 \times 5 \times 4 = 120$. In total there are $15 + 120 = 135$ possible three letter words.

17. **B.** Based on our understanding of inscribed angles, we know

$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F + \angle G + \angle H =$$

$$\frac{1}{2}(m\widehat{FG}) + \frac{1}{2}(m\widehat{GH}) + \frac{1}{2}(m\widehat{HE}) + \frac{1}{2}(m\widehat{EF}) + \frac{1}{2}(m\widehat{CD}) + \frac{1}{2}(m\widehat{DA}) + \frac{1}{2}(m\widehat{AB}) + \frac{1}{2}(m\widehat{BC}) =$$

$$\frac{1}{2}(m\widehat{FG} + m\widehat{GH} + m\widehat{HE} + m\widehat{EF}) + \frac{1}{2}(m\widehat{CD} + m\widehat{DA} + m\widehat{AB} + m\widehat{BC}) =$$

$$\frac{1}{2}(360°) + \frac{1}{2}(360°) = 360°$$

18. **E.** The first five terms of this sequence are $a_1 = 1, a_2 = 4, a_3 = 10, a_4 = 22, a_5 = 46$. So, the first four differences of consecutive terms are $a_2 - a_1 = 3, a_3 - a_2 = 6, a_4 - a_3 = 12, a_5 - a_4 = 24$. Noticing that these differences are forming a geometric sequence, we conclude that $a_n - a_{n-1} = 3(2)^{n-2}$. So, plugging in $n = 2019$ we find $a_{2019} - a_{2018} = 3(2)^{2017}$.

19. **D**. The graph of f^* and g^* is a geometric transformation of the graph of f and g. f^* and g^* are formed by applying a horizontal translation to the left by 3 units followed by a vertical and horizontal dilation by factor of $\frac{1}{2}$ to the graphs of f and g. The area of the region enclosed between the graphs is not affected by a horizontal translation. A dilation by a factor of $\frac{1}{2}$ does affect the area of the region. The area of the new region is $\left(\frac{1}{2}\right)^2$ times the area of the former region. The area of the new region is $\frac{1}{4}$ times 4 (the area of the old region). So, the area of the new region is 1.

20. **E**. $g(x) = f(x+1) - 1$ allows us to rewrite the expression all in terms of f.

$$g(1) + g(-3) + g(2) + f(-3) + 7$$
$$= f(2) - 1 + f(-2) - 1 + f(3) - 1 + f(-3) + 7$$

$$= f(2) + f(-2) + f(3) + f(-3) + 4$$
$$= f(2) - f(2) + f(3) - f(3) + 4$$
$$= 0 + 0 + 4 = 4$$

21. **D.** $\frac{3}{\log 5} - \frac{(\log_5 4)(\log_5 8)(\log_6 5)}{(\log_{10} 9)(\log_3 10)(\log_6 2)} = \frac{3}{\log 5} - \frac{(2\log_5 2)(3\log_5 2)(\log_6 5)}{(2\log_{10} 3)(\log_3 10)(\log_6 2)}$ $= \frac{3}{\log 5} - \frac{(2\log_5 2)(3\log_6 2)}{2\log_6 2} = \frac{3}{\log 5} - 3\log_5 2 = \frac{3}{\log 5} - \frac{3\log 2}{\log 5} = \frac{3(1-\log 2)}{\log 5} = \frac{3(\log 10 - \log 2)}{\log 5} = \frac{3(\log 5)}{\log 5} = 3$. No rounding is necessary.
22. **B.** Let the number picked by Zonshen be z. Richard's number will differ from Zonshen's number by less than 1 as long as he picks a number in the range $(z-1, z+1)$. This range has length 2. The range of numbers Richard may choose from has length 8. The probability Richard chooses a number from the interval $(z-1, z+1)$ is $\frac{2}{8} = \frac{1}{4}$.
23. **C.** If $x \geq 6$, then neither absolute value function has an effect. That is, for $x \geq 6$, $f(x) = \frac{x-3-1}{2x-12-4} = \frac{x-4}{2x-16} = \frac{x-4}{2(x-8)}$. There is no removable discontinuity in this portion of the domain. If $x \leq 3$, then the arguments of both absolute values will be negative and made positive. That is, for $x \leq 3$, $f(x) = \frac{-(x-3)-1}{-(2x-12)-4} = \frac{-x+2}{-2x+8} = \frac{x-2}{2(x-4)}$. There is no removable discontinuity in this portion of the domain. Finally, for $3 < x < 6$, the absolute value signs in the numerator will have no effect but the absolute value signs in the denominator will negate (making a negative positive) its argument. For $3 < x < 6$, $f(x) = \frac{x-3-1}{-(2x-12)-4} = \frac{x-4}{-2x+8} = \frac{x-4}{-2(x-4)}$. $f(4)$ is undefined, but for every x-value near $x = 4$, $f(x) = \frac{x-4}{-2(x-4)} = -\frac{1}{2}$. So, the point $\left(4, -\frac{1}{2}\right)$ is a removable discontinuity. $4 + \left(-\frac{1}{2}\right) = \frac{7}{2}$.
24. **B.** The lowest degree term in the polynomial will be $(x^6)^5 = x^{30}$. The highest degree term will be $(x^8)^5 = x^{40}$. Since all integral powers between 30 and 40 will also appear, there will be 11 terms in the expansion.
25. **D.** Let the point at the base of the 80 foot pole be called A, the point at the base of the 20 foot pole be called B, the point on the ground directly below where the cables meet be called C, and the height of the point at which the cable meet be called x. By similar triangles $\frac{x}{80} = \frac{BC}{BC+CA}$ and $\frac{x}{20} = \frac{CA}{BC+CA}$. Since $4\left(\frac{x}{80}\right) = \frac{x}{20}$, we see that $4\left(\frac{BC}{BC+CA}\right) = \frac{CA}{BC+CA}$. So, $4 \times BC = CA$. Substituting this back into one of the similar triangle equalities we find $\frac{x}{20} = \frac{4\times BC}{BC+4\times BC} = \frac{4\times BC}{5\times BC} = \frac{4}{5}$. Solving for x we arrive at $x = 16$.
26. **C.** Rearranged this equality is $(y-5)^2 - \frac{(x+2)^2}{4} = 1$. The asymptotes of this hyperbola pass through the point $(-2,5)$ with slopes of $\frac{1}{2}$ and $-\frac{1}{2}$. The asymptotes are $y = \frac{1}{2}(x+2) + 5$ and $y = -\frac{1}{2}(x+2) + 5$. The y-intercepts of these lines are $(0,6)$ and $(0,4)$. $|6-4| = 2$.

27. **B**. Adam paints $\frac{4}{3}$ houses each day and Alex paints $\frac{3}{4}$ houses each day, so together they paint $\frac{3}{4}+\frac{4}{3}=\frac{25}{12}$ houses each day. To paint 60 houses it will take $60\ houses \div \frac{25\ houses}{12\ days} = 28.8\ days.$
28. **A.** $f(x)=\sqrt{x^2-6x+9}=\sqrt{(x-3)^2}=|x-3|$ is not invertible. This is clearly the case since absolute value graphs do not pass the "horizontal line test" (e.g. $f(4)=f(2)$). $g(x)$ is the sum of two monotonically increasing functions and is therefore monotonically increasing as well. So, it is one-to-one and therefore invertible. $h(x)$ is even. The symmetry of even functions suggests this will not pass the horizontal line test. Sure enough, $h(-2)=h(2)$. The only invertible function give is $g(x)$.
29. **C.** Before solving, it should be noted that domain of the expression on the right side of the inequality is restricted to $\left(0,\frac{1}{e}\right)\cup\left(\frac{1}{e},\infty\right)$, so the solution to this inequality must exist entirely within this domain. Moving everything to one side of the inequality we get $\frac{1}{\ln x+1}-1\le 0$ which is equivalent to $\frac{-\ln x}{\ln x+1}\le 0$. The values that possibly form the endpoints of the intervals that are solutions to this inequality are the values of x that make the numerator or denominator zero. These are $x=\frac{1}{e}$ and $x=1$. For $x<\frac{1}{e}$, the numerator of the left side of the inequality will be positive and the denominator will be negative, so the inequality will be true. For $\frac{1}{e}<x<1$, both the numerator and denominator will be positive, so the inequality will not be satisfied. For $1<x$, the numerator will be positive, the denominator will be negative, and we will conclude that the inequality is satisfied. Note that at $x=1$, the inequality is also satisfied. So, the solution to the inequality will be the values of x such that x is within the initial domain restriction and $x<\frac{1}{e}$ or $x\ge 1$. This is $\left(0,\frac{1}{e}\right)\cup[1,\infty)$.
30. **E.** $\left(\frac{1+i}{2}\right)^{10}=\left(\left(\frac{1+i}{2}\right)^2\right)^5=\left(\frac{2i}{4}\right)^5=\left(\frac{i}{2}\right)^5=\frac{i^5}{2^5}=\frac{i}{32}.$